An optimization model of groundwater resources allocation for agriculture in well irrigation district of North China

Qian Huang 1,2,3, Shizhang Peng 1*, Zhendong Du 2 and Ningjiang Lu 2
1 State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, Nanjing, China. 2 Water Resources Research Institute of Shandong Province, Jinan, China. 3 Shandong Provincial Key Laboratory of Water Resources and Environment, Jinan, China.*e-mail: szpeng@hhu.edu.cn

Received 8 June 2012, accepted 6 October 2012.

Abstract
Due to excessive exploitation of groundwater resources to meet crop irrigation requirement, shallow groundwater table is falling at the rate of about 1m/y for the last 20 years in North China plain, which has caused many ecological and environmental problems. A kind of non-linear multi-year optimization model of groundwater resources allocation (MOGA) has been developed to decrease groundwater exploitation as well as keep high yield of crops at the same time. The model has been applied for agricultural water supply in Huantai County, a well known irrigation district in north China, and achieved good fruits. In the model, average crop yield in optimization period was maximized as objective function and crop optimal irrigation regime based on Jensen function were coupled in the constraints to construct a non-linear optimization model with reliable precision and practicability. According to the characteristics of precipitation in studied area, combination of three hydrological years, namely wet year (P = 25%), average year (P = 50%) and dry year (P = 75%), were presented as an optimization period in the model parameter set. Through four planned scenarios, optimized planting areas, irrigation quota and grain yields for different crops in three hydrological years could be calculated optimally, respectively, using LINGO optimization software. The result indicates that high food crop yield could be obtained with 80% of total available groundwater resources, almost same as crop yield when irrigating with total available groundwater. The optimization model and optimal scheme provide decision makers a theoretical way to implement deficit irrigation and manage groundwater resources in well irrigation district, coping with the problem of extensive exploitation of groundwater from the aspect of water resources allocation.

Key words: Optimization model, groundwater, well irrigation district, agriculture, North China, LINGO.

Introduction
North China, covering an area about 1.5 million km², has been reported 1 to produce more than one-fifth of the total state grain yield. In this region, the widely cultivated crops are winter wheat and summer maize, and winter wheat-summer maize double-cropping system is commonly adopted 2. The mean annual rainfall is around 600 mm and precipitation ranges from 100 to 180 mm, accounting for approximately 25-40% of winter wheat water requirement over the growing season. Therefore, water is often a limiting factor for crop growth in semi-arid or semi-humid regions and irrigation for crops is required in order to obtain high grain yield. In North China plain, due to the shortage of surface water, groundwater has been pumped irrationally for agricultural irrigation resulting to the decline rate of 0.5 ~1m of groundwater level per year 3. The major environmental pressures have an impact on the quantity of groundwater resources 4. In order to keep balance of groundwater exploitation and recharge, the agricultural water supply needs to decrease, nevertheless high crop yield is required to be obtained at the same time. So, it is substantially important to promote efficient use of water through better management of water resources, for social and economic sustainability in arid, semi-arid and semi-humid areas, under the conditions of severe water shortage 5,6. It has been proved by many experiments and researches that appropriate scheduling, which is always embedded in optimal allocation, can increase crop yields when irrigation water is insufficient 7,8. Much work has been done in optimized irrigation scheduling for different crops 9-14.

To maintain the balance of supply and exploitation of groundwater with scarce surface water resources and utilize water resources rationally, it is completely essential to implement multi-year optimal allocation of groundwater as well as make efficient use of irrigation water in well irrigated district. Compared to year-round optimal allocation of groundwater, the characteristic feature of groundwater storage and exploitation is considered, which can be less exploited in wet year and overexploited in dry year in certain extent without resulting in declining of groundwater level in a long time. An effect of approach for solving groundwater allocation issue during a period is presented to develop a multi-year optimization model of groundwater allocation (MOGA) in this paper.

The model MOGA applies system analysis and operational research to get the most appropriate allocation strategy of water resources through constructing objective function and relative constraints, which provides theoretic evidence and mathematic data support for supervisor to make decision in well irrigation...
district. In the model, complexity of water distribution for crops in various years is considered fully, thus it can solve the problem of optimal allocation of water resources in the area where irrigation is dependent on groundwater. With substantial merits in the model, a case in point is evaluated to use the model into practical application in Huantai County, a noted well irrigated district sited in north China.

**Material and Methods**

Before developing the optimal model, running operation mode of groundwater withdrawals and irrigation in well irrigated scheme should be considered fully, on which the modeling principle is based. Multi-year average recharge amount in a region is almost determined, but the groundwater withdrawal for irrigation in years varies substantially due to variation of precipitation, namely less than average water withdrawal amount in one wet year and more than that in one dry year. However, the impact to groundwater level of the exploited region would be seen after a long time. Thus, in the process of modeling, the amount of groundwater in well irrigated scheme should be considered as an underground reservoir, which can be regulated and distributed among years within a certain hydrological period, e.g. a combination of wet year, dry year and average year. In this period, the optimal objective function is to maximize the mean crop yield or net irrigation benefits in three years under the condition of keeping the balance of groundwater and subject to system constraints.

**Objective function of the optimization model:** A number of alternative objective function formulations is possible for the water allocation problem. In MOGA model, the objective is to maximize net irrigation benefit, i.e. multi-year average gross irrigation benefit minus cost of pumping groundwater, which can be expressed as:

Maximize \( F = F_m - F_g \). \( (1) \)

\[
F_m = \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{k} \epsilon_{ij} \cdot Y_{ij} \cdot A_i \cdot (\alpha_i, Y_{ij})
\]

\( (2) \)

\[
F_g = \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{k} (C_g \cdot Q_{ij} / \eta_g)
\]

\( (3) \)

where \( F = \) multi-year average net benefits of irrigation; \( F_m = \) multi-year average gross benefit of irrigation; \( F_g = \) multi-year average cost of pumping groundwater; \( n = \) the total year number of calculating period; \( k = \) the number of crop variety; \( \epsilon_{ij} = \) benefit-sharing coefficient in scheme \( i \); \( \alpha_i = \) price of crop \( i \); \( A_i = \) planting area of crop \( i \); \( Y_{ij} = \) potential yield for crop \( i \); \( \alpha_i = \) relative yield coefficient of crop \( i \), equaling the actual yield dividing \( Y_{i\text{max}} \); \( \eta_g = \) efficiency coefficient of irrigation water; \( C_g = \) irrigation cost by pumping groundwater; \( Q_{ij} = \) pumping groundwater amount.

In (3), irrigation cost is combined by three parts as following:

\[ C_g = C_{g1} + C_{g2} + C_{g3} \]

\( (4) \)

where \( C_{g1} = \) energy cost of pumping groundwater unit; \( C_{g2} = \) maintenance and management cost for pumping equipment and wells; \( C_{g3} = \) depreciation cost of well engineering.

For \( C_{g1} \), the energy cost of pumping groundwater is mainly power fee for engineering operation, responsible to the equipment pumping efficiency and height of dynamic water level. As water table will decline with pumping groundwater, the \( C_{g1} \) is, in general, a non-linear function of pumping rate of all active wells. However, dynamic water table nearly keep balanced after a temporary pumping according to groundwater pumping experiment in field, so we assumed that \( C_{g1} \) in the model is a constant for the specifically irrigated district without hovering in pumping time.

**Decision variables of the optimization model:** The decision variables include operating decision variables (irrigation quota of various crops), planting structure variables (optimal planting areas of crops) and crop yields. The decision variables in the optimal model are listed in Table 1. For the research region, the crops are mainly winter wheat, summer maize, fruit trees, vegetables and other small spot planting crops.

**Table 1. Decision variables in optimization model.**

<table>
<thead>
<tr>
<th>Crops</th>
<th>Wheat</th>
<th>Maize</th>
<th>Orchard</th>
<th>Vegetable</th>
<th>Others*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Irrigation area</td>
<td>A1</td>
<td>A2</td>
<td>A3</td>
<td>A4</td>
<td>A5</td>
</tr>
<tr>
<td>Irrigation quota</td>
<td>M1</td>
<td>M2</td>
<td>M3</td>
<td>M4</td>
<td>M5</td>
</tr>
<tr>
<td>Yield</td>
<td>Y1</td>
<td>Y2</td>
<td>Y3</td>
<td>Y4</td>
<td>Y5</td>
</tr>
</tbody>
</table>

Note: Other crops include cotton, soybeans, peanuts, yams, vegetables in greenhouse and so on.

**Constraints of the optimization model:** There are two groups of constraints. The first group of constraints ensure the total amount of groundwater pumped from each of scheme unit equaling to water amount used (if the well irrigated region is small and the hydraulic features and planting distribution are almost similar, the differences between units can be negligible, which are treated as one “big” unit). The second group ones considers planting customs and planting construction demand for specialized need, for instance, grain crops planting area for food security and so on.

a) Constraint of total area: \( \sum_{i=1}^{n} A_{ti} = A_t \) for \( t=1,n \) \( (5) \)

where \( A_t = \) total crops planting area in the \( t \) year; \( A_{ti} = \) planting area of crop \( i \).

b) Constraint of crop \( i \) water amount used: \( Q_{ti} = M_{ti} \cdot A_{ti} \) \( (6) \)

c) Constraint of relative crops yield and irrigation quota:

\[
\alpha_i = \frac{Y_{ij}}{Y_{i\text{max}}} = \left\{ \begin{array}{ll}
1 & M_{i\text{max}} \leq M_{i}\n \beta_{i0} + \beta_{i1} M_{i} + \beta_{i2} M_{i}^2 & M_i < M_{i\text{max}}
\end{array} \right.
\]

\( (7) \)

where \( M_{i\text{max}} = \) maximum irrigation quota under which crop \( i \) can be got potential yield, \( \alpha_i = \) relative yield coefficient, \( \beta_i = \) empirical regression coefficient; \( Y_{ij} = \) actual crop \( i \) yield. \( \beta_i \) is determined by irrigation experiment and crop optimal irrigation schedule. As far as economic crops are concerned, such as fruit trees and vegetables, sufficient irrigation is implemented, so relative yield coefficient is 1.

d) Constraint of total water amount available:

\[
\sum_{i=1}^{n} \sum_{j=1}^{k} Q_{ij} / \eta_g \leq \sum_{i=1}^{n} W_i
\]

\( (8) \)
where $W_t$ amount of groundwater available for the $t$ year.

e) Constraint of the planting proportion:
In research region, the planting area of food crops, vegetables, fruit trees and other crops should be needed to meet approximately proportion for domestic living demand and regional agricultural planning.

f) Non-negative constraints: Various types of decision variables must not be negative.

**Solution for model**: Because this is non-linear relationship between constraints and objective function in the model, optimal solution for the non-linear model is more complicated and difficult, compared to simplified linear ones. In this article, optimization software LINGO is used to optimize the highly non-linear procedure which has substantial powerful compiler “Global Solver” and strong optimization capability. Some good starting points are provided for the solution before running the procedure in LINGO software and global optimal solution is always achieved. The concerning approach can found in Optimization Modeling with LINGO\(^{17}\). Using LINGO makes the process of procedure easier and efficient.

**Case study**: The mathematical optimal model is applied to the typical well irrigated area, Huantai county located in north China, where shallow groundwater from the aquifer is single irrigation source for agriculture. In the passing three decades, groundwater level has been declined continuously overall \(^{18}\). Groundwater resources is optimally allocated through various crops during different hydrological years in this region by the MOGA model and provided effective utilization way and management method to solve the water insecurity issue.

Huantai County, covering 499 km\(^2\), is located in the center of Shandong Province in North China Plain (N36°512503-37°062003;E117°502003-118°102403) as can be seen in Fig. 1. It is noted by being the first county gaining average food crop yield (winter wheat and summer maize) more than 15 thousand kg per ha annually in north China with intensive cultivation. It has a high level of agricultural production for winter wheat and summer maize double–cropping system area. This region has annual average precipitation of 537 mm less than that of 600 mm in north China, and almost depends on exploiting shallow groundwater for agricultural irrigation, due to severe shortage of surface water resources. Because of over-exploitation in recent three decades, the average groundwater level of whole county has been continuously declining to nearly 12 m, compared to roughly 3 m in early 1980’s as seen in Fig. 2.

At present, the county has 25.8 thousand ha cultivated area and 13 thousand pumping wells, corresponding to 2 ha cultivated area irrigated by one well. The up to date issue is excessive wells and unreasonable allocation and controlling of groundwater resources in this region. To cope with the shortage of water resources, it is necessary to adopt water-saving agriculture countermeasures to achieve the highest possible increase in the water-use efficiency (WUE) of crops \(^{19}\), and water resources optimal distribution for irrigation is indispensable from point of view of water resources utilization strategy.

There are four scenarios to describe present and future possible conditions, in which irrigation water utilization coefficient and groundwater withdraw are considered differently.

1) **Hydrological optimal period of allocation.** According to annual precipitation from 1976-2008, wet year, average year and dry year, account for 35%, 30% and 35% of total precipitation year number respectively in Huantai county. That is to say, the above mentioned three precipitation frequency account for one third of total year roughly. Therefore, the hydrological combination of wet year (P = 25%), average year (P = 50%) and dry year (P = 75%) in the model can represent multiyear precipitation characteristic locally. We apply the three year as optimal period of groundwater resources allocation.

2) **Planting area of crops.** Considering of food security and regional agricultural planning, food crop (winter wheat and summer maize) planting area is not less than 85% of cultivated area, vegetable not more than 7% and fruit tree area not more than 10%.

3) **Potential yield of various crops.** The possible maximum or potential yields per unit area of various crops are seen in Table 2. Yield of winter wheat is 8250 kg per hm\(^2\) under sufficient irrigation conditions.

**Table 2.** The potential yield per unit area of various crops in Huantai County.

<table>
<thead>
<tr>
<th>Crop variety</th>
<th>Wheat</th>
<th>Maize</th>
<th>Fruit tree</th>
<th>Vegetable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Potential yield(kg/hm(^2))</td>
<td>8250</td>
<td>9750</td>
<td>30000</td>
<td>60000</td>
</tr>
</tbody>
</table>
which can meet water demand of maximum evapotranspiration about 550 mm during wheat growth season. Yield of summer maize is 9750 kg per hm² with maximum evapotranspiration of 430 mm. As to economic crops, such as fruit trees and vegetables, sufficient irrigation is always implemented even though encountering drought climate, so potential yield are always gained.

(4) Agricultural water amount available. The average annual groundwater amount available is 142.2 million m³ in Huantai County, in which main proportion comes from local precipitation.

Results and Discussion
According to current and future water efficiency of irrigation and water supply amount in planning optimal period, four scenarios for optimization were designed.

(1) The first scenario is to allocate groundwater resources optimally under current irrigation condition with irrigation water utilization coefficient of 0.75 and total groundwater amount available of 142.2 million m³ for every year, namely wet year, average year and dry year.

(2) The second scenario is to allocate groundwater resources optimally under improved future irrigation technology with irrigation water utilization coefficient of 0.85 and total groundwater amount available of 142.2 million m³ for every year.

(3) The third scenario is to allocate groundwater resources optimally under improved future irrigation technology with irrigation water utilization coefficient of 0.85 and 90% of total groundwater amount available for every year.

(4) The fourth scenario is to allocate groundwater resources optimally under improved future irrigation technology with irrigation water utilization coefficient of 0.85 and 80% of total groundwater amount available for every year.

The optimal procedure under LINGO language environment is programmed to solve the problem. We take the fourth scenario as an instance to find optimization solution and demonstrate the result. The optimal planting area is with wheat 23.5 thousand hm², maize 23.5 thousand hm², fruit trees 1.8 thousand hm² and vegetables 0.5 thousand hm² (Table 3). The optimal crop net irrigation quota, allocation of water and yield of winter wheat are in Table 4. Yields of winter wheat in wet year, average year and dry year, respectively, accounts for 88.4%, 88.9% and 90.9% of potential yields and mean yield for three years is 7380 kg per hm², accounting for 89.4% of potential yield. Yields of summer maize in wet year, average year and dry year can all equal potential yield 9750 kg per hm².

Gross irrigation quota for winter wheat in three years are respectively 253, 265 and 334 mm and average gross irrigation quota is 284 mm, namely 2840 m³/hm². According to the concerning research, farmers in this region usually apply 4-6 times irrigation about 3000-4500 m³/hm² through border irrigation during the winter wheat growing season for a little higher yield. Gross irrigation quota for summer maize in three years are respectively 120, 181 and 206 mm and the potential high yields can be gained in three years (Table 5). By the optimization model, if measure of water allocation is adopted, more than 28.45 million m³ groundwater is saved every year with substantially high yield that can ensure regional food security.

Under the fourth scenario, the optimal mean gross irrigation benefit is 487.8 million Yuan, the mean irrigation cost 16.96 million Yuan and groundwater for irrigation 113.79 million m³ as can be seen in Table 6. If net irrigation benefit in three years is compared each other, it is easy to find that there is hardly any difference or equal each year when the unique optimum objective function value is got. What’s more, the solution also means that high crop yield is got and regional groundwater level is promoted up to 1.04 m with significant environmental benefit if agricultural water supply amount decreases 20% of total groundwater amount available.

**Table 3. Optimal crops area under the fourth scenario.**

<table>
<thead>
<tr>
<th>Crops</th>
<th>Winter</th>
<th>Maize</th>
<th>Fruit tree</th>
<th>Vegetable</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area (10⁴hm²)</td>
<td>2.35</td>
<td>2.35</td>
<td>0.18</td>
<td>0.05</td>
<td>2.58</td>
</tr>
</tbody>
</table>

**Table 4. Optimal winter wheat irrigation quota, allocation amount of water and yield.**

<table>
<thead>
<tr>
<th>Hydrological year of irrigation</th>
<th>P=25%</th>
<th>7290</th>
<th>215</th>
<th>253</th>
<th>5889</th>
</tr>
</thead>
<tbody>
<tr>
<td>(η = 0.85) P=50%</td>
<td>7335</td>
<td>225</td>
<td>265</td>
<td>6234</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7500</td>
<td>284</td>
<td>334</td>
<td>7851</td>
<td></td>
</tr>
</tbody>
</table>

**Table 5. Optimal summer maize irrigation quota, allocation amount of water and yield.**

<table>
<thead>
<tr>
<th>Hydrological year of irrigation</th>
<th>P=25%</th>
<th>9750</th>
<th>102</th>
<th>120</th>
<th>2817</th>
</tr>
</thead>
<tbody>
<tr>
<td>(η = 0.85) P=50%</td>
<td>9750</td>
<td>154</td>
<td>181</td>
<td>4254</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9750</td>
<td>175</td>
<td>206</td>
<td>4834</td>
<td></td>
</tr>
</tbody>
</table>

**Table 6. Optimal irrigation benefit and cost under the fourth scenario.**

<table>
<thead>
<tr>
<th>Hydrological year</th>
<th>Gross irrigation benefit(million Yuan)</th>
<th>Irrigation cost (million Yuan)</th>
<th>Net irrigation benefit (million Yuan)</th>
<th>Irrigation water amount (million m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P=25%</td>
<td>485.80</td>
<td>13.87</td>
<td>471.93</td>
<td>93.07</td>
</tr>
<tr>
<td>P=50%</td>
<td>487.03</td>
<td>16.73</td>
<td>470.30</td>
<td>112.41</td>
</tr>
<tr>
<td>P=75%</td>
<td>490.76</td>
<td>20.28</td>
<td>470.49</td>
<td>135.90</td>
</tr>
<tr>
<td>Average</td>
<td>487.87</td>
<td>16.96</td>
<td>470.91</td>
<td>113.79</td>
</tr>
</tbody>
</table>
Conclusions
The non-linear multi-year optimization model of groundwater resources allocation (MOGA) has been proved effective to solve the issue of allocation of groundwater resources for agriculture irrigation. The groundwater would be distributed reasonably in different hydrological years and crop growth stages that proposes a feasible way of dealing with sustainable utilization of groundwater in well irrigation district. With the presented model and overall agricultural water-saving technology, high yield of crops and economic benefit can be all gained in Huantai County when decreasing 20% of the available groundwater supply. It provides the water management department with theoretical basis for decisions making.

Acknowledgements
The study presented here is supported by National High Technology Research and Development Program (863 project), with the project number of 2006AA100222.

References