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Relations between solid fractal dimension and some physical properties of soils formed over alluvial and colluvial deposits

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Abstract

Results of many studies showed that fractal dimension of particle-size distribution (PSD) was useful to evaluate the relationship between soil texture and its associated properties. In this study we evaluated relationship between solid fractal dimension (Ds) and some of the physical properties of soils (Mollic Ustifluvent, Typic Ustifluvent and Typic Ustorthent) formed over alluvial and colluvial deposits in North Central Anatolia of Turkey. An area of nine ha was sampled intensively, collecting 168 disturbed and undisturbed soil samples. Ds for each sample was calculated from particle size distribution data. Soils were further analyzed for texture, organic matter content (OM), bulk density (BD), saturated hydraulic conductivity (Ks), field capacity (FC) and van Genuchten parameters (α and n). Relations between Ds and soil physical properties were analyzed by regression technique. Ds ranged from 2.64 to 2.91, finer textures yielding greater Ds values. In contrast to significant and positive correlations occurred between Ds and each of clay, Ks, FC and α, correlations between Ds and each of sand, silt and n were negative. The results showed that Ds has significant relations with soil physical properties and may be used as an integrating index in modeling studies.

Key words: Particle size distribution, soil texture, fractal dimension, hydraulic conductivity, van Genuchten parameters, alluvial, colluvial.

Introduction

Soil is consisting of numerous particles that differ in density, shape, size, etc. Interaction of these particles with each other and their arrangements in the space determines the soil structure and pore-solid fractal geometry. Fractal geometry and fractal dimension [1] has been used to identify heterogeneity occurred in natural processes, such as soil physical systems [2].

Fractal dimensions have been increasingly used to describe physical properties of porous media. There are numbers of published studies related to characterization of soil physical properties including particle-size distribution, pore-size distribution and aggregate-size distribution using their fractal dimensions. Rieu and Sposito [3] stated that precise measurement of soil physical properties and fractal parameters is necessary for use of fractal concept in soil studies.

Results of many studies showed that fractal dimension of particle-size distribution (PSD) was useful to evaluate the relationship between soil texture and its associated properties [4, 5]. PSD can be represented as a parameter by Ds in modeling studies. Many studies have been conducted to evaluate the function of Ds as an interpreting factor that may facilitate parameter reduction in modeling studies. Most of these studies have been focusing on modeling [6, 7], fragmentation [2, 8-10], scaling[11, 12], estimation of soil water retention curve and hydraulic conductivity [13-22], aggregate stability [23] and soil erodibility and desertification [24-26]. More research is needed to better understand the inherent relation between Ds and associated soil processes.

The objective of this study was to analyze the functional relationship between Ds and soil physical properties such as textural components (sand, silt and clay), field capacity (FC), saturated hydraulic conductivity (Ks) and van Genuchten water retention parameters α and n (-) in an alluvial-colluvial area (Mollic Ustifluvent, Typic Ustifluvent and Typic Ustorthent).

Theory

The number-size relationship given by Mandelbrot [1] is the fundamental concept and defined as $N(r) = kr^{-D}$ (Eq. 1). N(r) is the number of elements with length equal to r, k is the number of initiators with unit length and D is the fractal dimension [15, 27]. Fractal dimension of objects, with a regular shape but different characteristic size, l, can be related to number-size concept using Eq. (1) and given as $N(l) = \propto l^{-D}$ (Eq. 2). Fractal dimension for PSD was generally calculated using number-mass approach [8, 13, 27] as given in $M \propto r^{Ds}$, $Ds \leq d$ (Eq. 3), where d is topological dimension (1, 2, 3), Ds is the fractal dimension of solid or mass quantifying space filling properties of solid mass, M, with radius, r [8, 15, 27].

The most convenient method to determine fractal dimension is to use mass-based equation, with the following form: $M(r < R)/M_r = (R/R_{upper})^{3-Ds}$ (Eq. 4), where M(r<R) is the mass of soil including particles with a radius smaller than R, R_{upper} is upper size limit for fractal (2 mm for soil), M_T is the total mass of soil and Ds is the solid fractal dimension. Measurement of upper boundary of mass fractionation can be obtained by sieving and lower boundary by settling time [8, 13, 27].

Material and Methods

Material: Samples were taken from a nine-ha area of alluvial-colluvial soil near Tokat city in North-Central Anatolia of Turkey. The area was divided into 50 m x 50 m square-grids. In addition to grid intersects, two edges of the area were sampled at 25 m distances making 59 samples, and then 13 fine-transects with sampling distance of 1, 2, 3, 5, 10, 20, 35 and 50 m were randomly placed in north-south and west-east directions, superimposing the lines between the intersects. This sampling pattern was chosen for potential spatial research and analysis. Therefore, total of 168 soil samples, disturbed and undisturbed for each, were taken from 0-30 cm depth of top soil.

Soil samples, air-dried and sieved using 2-mm sieve, were analyzed for textural components by Bouyoucos method [28] and for organic matter (OM) by Walkley-Black procedure [29]. Undisturbed soil samples were taken by core method [30]. Bulk densities (BD) were calculated using weights of oven dried soil samples and core volumes. Saturated hydraulic conductivity (Ks) was measured with a constant head permeameter [31]. Water retention curves and field capacity (FC) were obtained using pressure plate apparatus [32] measuring volumetric water content at -0.01, -0.02, -0.033, -0.05, -0.075, -0.1, -0.3, -0.5, -0.7 and -1 MPa soil water pressures. Van Genuchten parameters α (inverse of air entry value) and n (slope of water retention curve) were obtained experimentally using water retention data with inverse function of RETC computer program.

Estimating the fractal dimension of soil particle-size distribution (Ds): In this study, mass-size data, PSD, that was determined by hydrometer and sieve analyses as described by Gee and Bauder [28] were used with Eq. (4). To obtain Ds in Eq. (4), log $(M(r<R)/M_T)$ was regressed with log (R/R_{upper}). Then, Ds values were calculated from the slope of the resultant regression lines.

Statistical analysis: Descriptive statistics (mean, range, standard deviation, coefficient of variation (CV), skewness and kurtosis) were calculated for Ds and soil properties [sand, silt, clay, organic matter (OM), bulk density (BD), saturated hydraulic conductivity (Ks), field capacity (FC) and van Genuchten parameters α and n] using SPSS v13 program. Mean values of parameters having the same Ds values are used for calculations. The functional relationship between parameter Ds and each of sand, silt, clay, Ks, FC, α and n was evaluated by regression technique. Some outliers on scatter plots were discarded to improve the regression fit. Coefficient of determination (R^2), mean squared error (MSE), and significance of overall regression (P) were considered to evaluate quality of regression modeling [33].

Results and Discussion

Soil characteristics: Textures of the soils studied varied from clay to sandy clay loam. Clay was the most and silt the least variable parameter among the soil textural separates, and α was the most and Ds the least variable parameter among the soil properties studied (Table 1). In general, except α, the soil variables exhibited a normal-like distribution with relatively small values of skewness and kurtosis. The van Genuchten parameters α and n were right-skewed, indicating existence of some high extreme values, and clay content had flatter distributions with relatively short tails.

Solid fractal dimension (Ds): Finer textured soils are expected to have greater Ds values due to the complete fragmentation. Su *et al.* [26] found that Ds was strongly and positively associated with clay content and silt content. In addition, Tyler and Wheatcraft [34] reported that finer textured soils possessed Ds values close to 3.0. In a study by Gimenez *et al.*[15] values of Ds ranged from 2.75 to 2.99. In our study, Ds ranged from 2.64 to 2.91, greater Ds value occurring for clay and smaller for sandy clay loam.

Relationship between soil textural components and Ds: The relation between clay content and Ds is depicted in Fig. 1 in which Ds values shown to be strongly associated with clay content (MSE 2.0×10^{-4} and $P<1.0\times10^{-4}$). This observation is in agreement with results reported by Tyler and Wheatcraft [15] and Ersahin *et al.* [27].

Behavior of Ds as a function of sand content is illustrated in Fig. 2. The first degree regression equation satisfactorily described the relation between the two attributes (MSE and P-values of the regression modeling were $3.0\times10^{-4}$and $P<1.0\times10^{-4}$, respectively) (Fig. 2). In contrast to clay content, sand content had a very strong negative effect on Ds. This suggested that increasing sand-size particles decreased the goodness of fit due to the relative abundance of particles with bigger diameters among the others having different prevalence.

There was moderate, negative and linear relation between silt content and Ds (Fig. 3). The results of MSE of 1.3×10^{-4} and $P<1.0\times10^{-4}$ were acceptable (Fig. 3). The scatter plot of Ds vs. silt content indicates that the relation between the two attributes was stronger for the silt content < 32%.

Table 1. Summary statistics of soil physical properties.

Soil Property	Mean	Minimum	Maximum	[a]St. Dev.	[a]C.V.(%)	Kurtosis	Skewness
Clay (%)	33.1824	15.1563	50.2083	9.2539	27.888	-1.257	-0.1103
Silt (%)	31.806	23.4375	42.5	3.6332	11.423	0.088	0.0034
Sand (%)	35.0117	20	57.5	8.2231	23.4868	-0.7041	0.3705
[a]OM (%)	1.7216	0.7	3.1	0.5003	29.0618	-0.5637	0.2822
[a]BD (Mg m^{-3})	1.3526	1.1276	1.6689	0.1076	7.9553	-0.0896	0.4385
[a]Ks (ms^{-1})	1.08×10^{-5}	4.81×10^{-6}	1.79×10^{-5}	2.78×10^{-6}	25.74	-0.4184	0.3319
[a]FC (%)	30.60	21.43	46.98	0.0446	14.5731	0.2778	0.4788
[a]α (cm^{-1})	0.0155	0.0063	0.0369	0.0056	36.0759	1.8550	1.1801
[a]n (-)	1.2944	1.174	1.47	0.0557	4.3003	0.7305	0.8075
[a]Ds	2.8326	2.6363	2.9097	0.0489	1.7264	0.4171	-0.7850

[a]: St. Dev., Standard deviation; C.V., coefficient of variance; OM, soil organic matter content (%); BD, soil bulk density (Mg m^{-1}) ; Ks, saturated hydraulic conductivity; FC, field capacity (%); á, inverse of air entry value; n, van Genuchten slope parameter; Ds, fractal dimension of solids

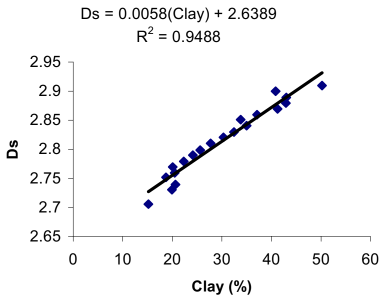

Figure 1. Solid fractal dimension, Ds, as a function of clay content. Each point represents a Ds vs. mean of corresponding clay contents.

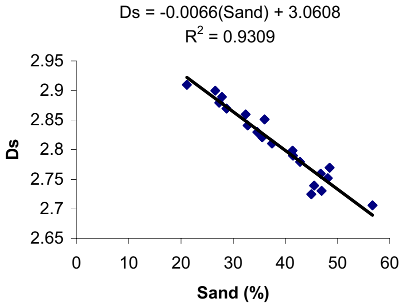

Figure 2. Solid fractal dimension, Ds, as a function of sand content. Each point represents a Ds vs. mean of corresponding sand contents.

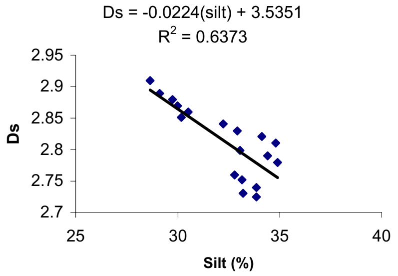

Figure 3. Solid fractal dimension, Ds, as a function of silt content. Each point represents a Ds value vs. mean of corresponding silt contents.

Relationship between soil hydraulic properties and Ds: Surprisingly, a moderate positive linear relation occurred between saturated hydraulic conductivity (Ks) and Ds (MSE of 0.0007 and a $P<1.0x10^{-4}$) (Fig. 4). This was attributed to the fact that the Ks was measured with disturbed soil samples passed through 2-mm screen and packed for field bulk density. Increasing clay content resulted in formation of more aggregates that behaved as sand particles, rendering increase in Ks. The strong positive relation between clay content and Ds resulted in that Ds represented influence of clay content on Ks. Measurement of Ks at field condition, using distributed soil samples instead, would be much more beneficial for further studies.

A scatter plot and regression equation referring relationship between FC and Ds are given in Fig. 5, where a second degree polynomial relation satisfactorily described the relation between the two attributes. MSE and P values of the regression modeling were $7.0x10^{-4}$ and $<1.0x10^{-4}$, respectively. Increase in self similarity in PSD strongly affected FC that may be attributed to the self-similarity in the pore-size distribution resulted in by Ds, greater Ds being associated with greater self-similarity in pore-size.

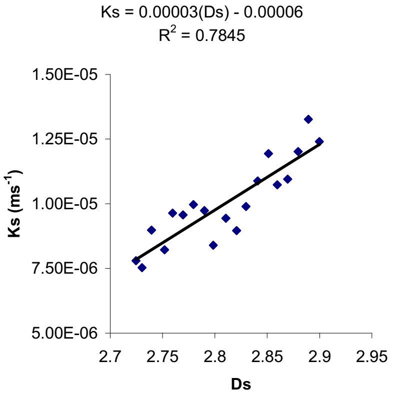

Figure 4. Values of solid fractal dimension, Ds, as a function of saturated hydraulic conductivity (Ks). Each point represents an Ds value vs. mean of corresponding values of Ks.

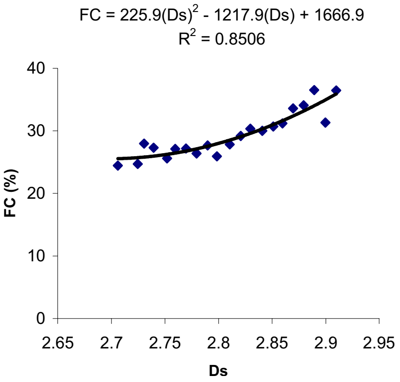

Figure 5. Solid fractal dimension, Ds, as a function of field capacity (FC). Each point represents a Ds value vs. mean of corresponding FC.

Relationship between van Genuchten parameters (α and n) and Ds: A strong, positive and second degree polynomial relation was observed between α and Ds (Fig. 6) while the relation between n and Ds was linear and negative (Fig. 7). MSE and P values of the regression models were $3.0x10^{-4}$ and $<1.0x10^{-4}$ for α parameter and $6.0x10^{-4}$ and a $P<1.0x10^{-4}$ for n parameter, respectively.

Parameter α is associated with the air entry value of the soils. High values of α are associated to fine textured soils. In this study a highly significant positive relation occurred between α and Ds, expectedly. Contrary to α, greater values of n (as absolute values) are associated to coarser textured soils, which usually possess smaller values of Ds. Parameter n is the slope of water retention curve at its steepest neighborhood. In coarse textured soils, high uniformity in the pore-size results in emptying the majority of the pores in a narrow range of water potential yielding steeper slope that possesses a greater value of n. Compared with curve-linear relation between α and Ds, a linear relationship occurred between n and Ds. The relation is highly significant. Ds could describe the 94% of the variability in α and 88% of the variability in n, which would be highly useful in the modeling of hydraulic conductivity in soils.

Conclusions

Ds significantly positively correlated with soil clay content, Ks, FC and α, despite significantly negatively correlated with sand

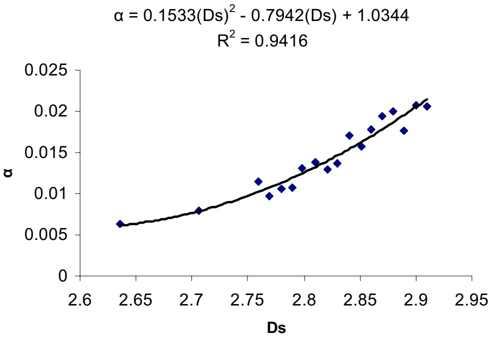

Figure 6. Solid fractal dimension, Ds, as a function of α. Each point represents a Ds value vs. mean of corresponding α.

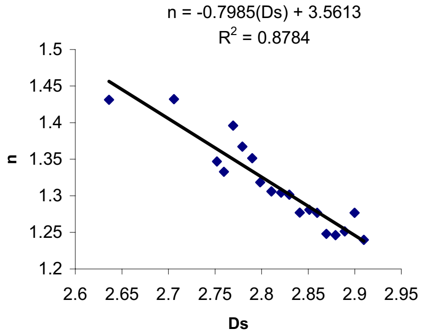

Figure 7. Solid fractal dimension, Ds, as a function of n. Each point represents a Ds value vs. mean of corresponding n.

content, silt content and n. Finer textured soils were associated with greater Ds. Soil parameters described were significantly positively or negatively correlated to Ds. Regression equations of Ds and soil parameters satisfactorily represented observed data. It can be concluded that Ds can be used to describe soil physical parameters or vice versa. Because of the strong and significant relations between Ds and soil parameters studied, Ds can be used in modeling studies to estimate soil physical parameters. More research should be conducted to determine boundaries and limitation on the use of Ds for development of regression and pedo-transfer functions.

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